Relative Expressive Power of Downward Fragments of Navigational Query Languages on Trees and Chains

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Overview

Introduction

Summary of the results

The boolean collapse to $\mathcal{N}()$

The collapse of $\cap$ and $\setminus$

Concluding remarks
Querying graphs

- Alice is the mother of Bob.
- Bob is the father of Eve.
- Eve works with Craig.
- Craig is the friend of Faythe.
- Faythe is the friend of Dan.
- Dan is the mother of Alice.
Querying graphs

\[ \pi_1[\text{motherOf}] \circ [\text{motherOf} \cup \text{fatherOf}]^+ \]
Querying graphs

\[ \pi_1[\text{friendOf} \setminus \text{worksWith}] \]
Navigational Expressions

\[ e := \emptyset \mid \text{id} \mid \ell \text{ (for } \ell \text{ an edge-label)} \mid e \circ e \mid e \cup e \mid [e]^+ \mid \pi_1[e] \mid \pi_2[e] \mid \overline{\pi}_1[e] \mid \overline{\pi}_2[e] \mid e \cap e \mid e \setminus e \]

**Question**
Are all these operations necessary?
How does each operator influence expressive power?

**Preliminary answer**
We can use basic rewriting:

\[ e_1 \cap e_2 = e_1 \setminus (e_1 \setminus e_2) \]
\[ \pi_i[e] = \overline{\pi}_i[\overline{\pi}_i[e]] \]
\[ \overline{\pi}_i[e] = \text{id} \setminus \pi_i[e] \]
Problem statement

We start with \(\{\emptyset, \text{id}, \cup, \circ\}\) and edge-labels

- Add any subset \(\mathcal{F}\) of \(\{\pi, \bar{\pi}, +, \cap, \setminus\}\),
  We denote the resulting query language by \(\mathcal{N}(\mathcal{F})\)
- Compare the expressive power of resulting languages
- Graphs: already fully studied by Fletcher et al.
- Trees: a few results are known (XML)

Definition
Let \(\mathcal{F} \subseteq \{+, \pi, \bar{\pi}, \cap, \setminus\}\).
\(\mathcal{F}\) is the superset of \(\mathcal{F}\) obtained by “basic rewriting”.

Example
\(\{\pi, \setminus\} = \{\pi, \bar{\pi}, \cap, \setminus\}\)
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Results on trees and chains: the main results

- On labeled trees, we can do without $\cap$ and $\\setminus$:
  \[ \mathcal{N}(\mathcal{F}) \preceq_p \mathcal{N}(\mathcal{F} \setminus \{\cap, \\setminus\}) \]

- For boolean queries:
  - On unlabeled trees, only $\pi$ adds expressive power:
    \[ \mathcal{N}(+, \pi, \cap) \preceq_b \mathcal{N}(\cdot) \]
  - On labeled chains, we can do without $\pi$:
    \[ \mathcal{N}(+, \pi) \preceq_b \mathcal{N}(\mathcal{F} \setminus \{\pi\}) \]
Results on trees and chains

<table>
<thead>
<tr>
<th></th>
<th>Boolean queries</th>
<th>Trees</th>
<th>Path queries</th>
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<tbody>
<tr>
<td><strong>Chains</strong></td>
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<tr>
<td>Labeled</td>
<td>$\mathcal{N}(\pi)$</td>
<td>$\mathcal{N}(+,\pi)$</td>
<td>$\mathcal{N}(+,\pi)$</td>
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<td></td>
<td>$\mathcal{N}(\pi, \cap)$</td>
<td>$\mathcal{N}(+, \pi, \cap)$</td>
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<tr>
<td>Unlabeled</td>
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The boolean collapse to \( \mathcal{N}(\cdot) \) on unlabeled trees

**Theorem**

Let \( \mathcal{F} \subseteq \{+ , \pi , \cap \} \). On unlabeled trees we have \( \mathcal{N}(\mathcal{F}) \preceq_b \mathcal{N}(\cdot) \).

**Definition (homomorphism)**

A mapping \( h : \mathbb{N}_1 \rightarrow \mathbb{N}_2 \) is a homomorphism from \( G_1 = (\mathbb{N}_1, E_1) \) to \( G_2 = (\mathbb{N}_2, E_2) \) if \( (m, n) \in E_1 \) implies \( (h(m), h(n)) \in E_2 \).

**Proposition**

The language \( \mathcal{N}(+, \pi, \cap) \) is closed under homomorphisms: if there is a homomorphism \( h \) from \( G_1 \) to \( G_2 \), then \( h(e(G_1)) \subseteq e(G_2) \).
Proof: $\mathcal{N}(+, \pi, \cap)$ cannot distinguish trees from chains.
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- Provide homomorphism from tree $\mathcal{T}$ to chain $\mathcal{C}$
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- Provide homomorphism from tree $\mathcal{T}$ to chain $\mathcal{C}$
- Provide homomorphism from chain $\mathcal{C}$ to tree $\mathcal{T}$
Proof: $N(+, \pi, \cap)$ can only query on depth

Conclusion

Even $N(+, \pi, \cap)$ can only express queries of the form:

The height of the tree is at least $k$ ($= \ell^k = \underbrace{\ell \circ \ldots \circ \ell}_{k \text{ terms}}$)

Theorem

Let $\mathcal{F} \subseteq \{+, \pi, \cap\}$. On unlabeled trees we have $N(\mathcal{F}) \preceq_b N()$. 


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Removing $\cap$ and $\setminus$ from simple queries

Example

$$[\ell^3]^+ \cap [\ell^7]^+ = [\ell^{21}]^+$$

$$[\ell^3]^+ \setminus [\ell^7]^+ = (\ell^3 \cup \ell^6 \cup \ell^9 \cup \ell^{12} \cup \ell^{15} \cup \ell^{18}) \circ ([\ell^{21}]^+ \cup \text{id})$$

Basic observations

- Expressions in $\mathcal{N}(+) \text{ are regular path queries}$
- Regular languages (expressions) are closed under $\cap$ and $\setminus$

Question

How to generalize to $\pi$ and $\overline{\pi}$?
Definition (condition automaton)

A condition automaton is a 7-tuple \( \mathcal{A} = (S, \Sigma, C, I, F, \delta, \gamma) \).

![Diagram of condition automata]
Semantics of condition automata

- Take a path in automaton from initial to final state
- Map to a path in a tree from $m$ to $n$ with equal labeling
- State $s$ maps to node $k$: $k$ satisfies the conditions of $s$
Semantics of condition automata

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Condition automata and navigational expressions

**Proposition**

Let $\mathcal{F} \subseteq \{+, \pi, \overline{\pi}\}$. The class of condition automata specified for $\mathcal{N}(\mathcal{F})$ in the following table is path-equivalent with $\mathcal{N}(\mathcal{F})$.

<table>
<thead>
<tr>
<th>Navigational language</th>
<th>Class of condition automata</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{N}()$</td>
<td>${+, \pi, \overline{\pi}}$-free and acyclic.</td>
</tr>
<tr>
<td>$\mathcal{N}(\pi)$</td>
<td>${+, \overline{\pi}}$-free and acyclic.</td>
</tr>
<tr>
<td>$\mathcal{N}(\pi, \overline{\pi})$</td>
<td>${+}$-free and acyclic.</td>
</tr>
<tr>
<td>$\mathcal{N}(\pi)$</td>
<td>${\pi, \overline{\pi}}$-free.</td>
</tr>
<tr>
<td>$\mathcal{N}(\pi, \overline{\pi})$</td>
<td>${\overline{\pi}}$-free.</td>
</tr>
<tr>
<td>$\mathcal{N}(\pi, \pi)$</td>
<td>no restrictions.</td>
</tr>
</tbody>
</table>
Condition automata and intersect

- Basically: use cross-product construction
- Take care of id-transitions by first removing them

Proposition

*Condition automata are closed under* $\cap$. 
Condition automata and intersect

- Basically: use cross-product construction
- Take care of \text{id}-transitions by first removing them

\begin{center}
\begin{tikzpicture}

\node[state,initial] (q1) at (0,0) {$q_1$};
\node[state] (q2) at (-2,-2) {$q_2$};
\node[state] (q3) at (2,-2) {$q_3$};
\node[state] (q4) at (-2,-4) {$q_4$};

\path[->]
(q1) edge node{$\ell$} (q2)
(q1) edge node{$\ell$} (q3)
(q2) edge node{id} (q4)
(q3) edge node{id} (q4)
(q2) edge node{id} (q1)
(q3) edge node{id} (q1);

\node at (0,-1.9) {$\{c_1\}$};
\node at (-2,-3.1) {$\{c_2\}$};
\node at (2,-3.1) {$\{c_3\}$};
\node at (-2,-5.1) {$\{c_4\}$};

\end{tikzpicture}
\end{center}

Proposition

\textit{Condition automata are closed under} $\cap$. 
Condition automata and difference

- Difference: in terms of ∩ and complement: $S \setminus T = S \cap \overline{T}$
- In our setting: restrict $T$ to the downward complement $T_{\downarrow}$

Definition (deterministic condition automaton)

- For each node $n$: there exists exactly one initial state $s$ such that $n$ satisfies $s$.
- If node $n$ satisfies state $q$, then, for each edge $(n, \ell, m)$ there exists exactly one transition $(q, \ell, p)$ such that $p$ satisfies $m$. 

```
q1 -------\[
 q2 \quad \ell \quad q3
     \ell

\{\pi_2[\ell]\}

\{\pi_2[\ell]\}

p1 -------/[
```


Condition automata and downward complement

- Downward complement of deterministic condition automaton
  
  *Swap the final states*

- Conclusion: if we can construct a deterministic condition automaton, then condition automata are closed under \(\setminus\)

**Proposition**

*For every condition automaton there is a path-equivalent deterministic condition automaton.*

In this construction \(\overline{\pi}\) is introduced if \(\pi\) was already used.

**Theorem**

*On labeled trees we have* \(\mathcal{N}(\mathcal{F}) \preceq_p \mathcal{N}(\mathcal{F} \setminus \{\cap, \setminus\})\).*
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Conclusions and Future Work

- Full characterization of the expressive power of downward navigational expressions
  - On trees and on chains
  - For boolean queries and path queries

- Typical non-downward operators are omitted
  - Next: we also include converse and diversity
  - We have some initial results
Proposition (Fletcher et al. ICDT’11)

Let $\mathcal{F}_1, \mathcal{F}_2 \subseteq \{+, \pi, \bar{\pi}, \cap, \setminus\}$.

- **Labeled graphs:**
  - $\mathcal{N}(\mathcal{F}_1) \preceq_b \mathcal{N}(\mathcal{F}_2)$: if $\mathcal{F}_1 \subseteq \mathcal{F}_2$.

- **Unlabeled Graphs:**
  - $\mathcal{N}(\mathcal{F}_1) \preceq_p \mathcal{N}(\mathcal{F}_2)$: if $\mathcal{F}_1 \subseteq \mathcal{F}_2$.
  - $\mathcal{N}(\mathcal{F}_1) \preceq_b \mathcal{N}(\mathcal{F}_2)$: if $\mathcal{F}_1 \subseteq \mathcal{F}_2$ or if $\mathcal{F}_1 \subseteq \{\pi\}$ and $\mathcal{F}_2 = \mathcal{F}_1 \cup \{+\}$.