Relative Expressive Power of Downward Fragments of Navigational Query Languages on Trees and Chains

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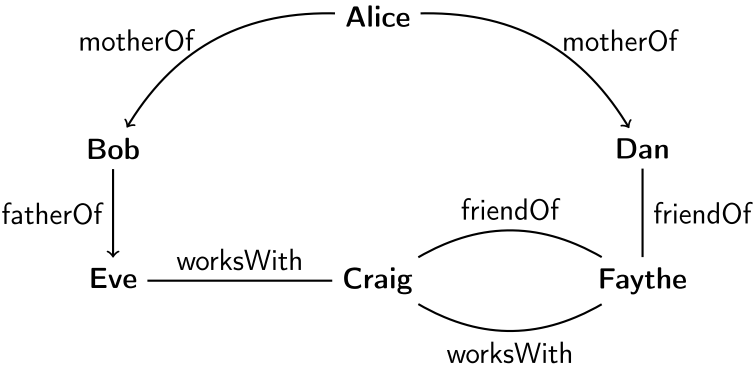
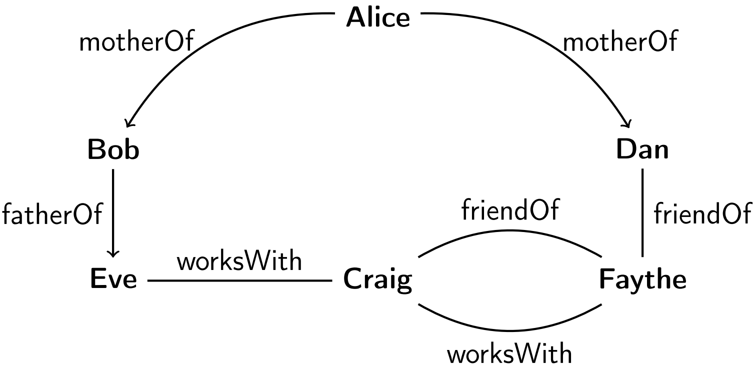
Overview

Querying graphs



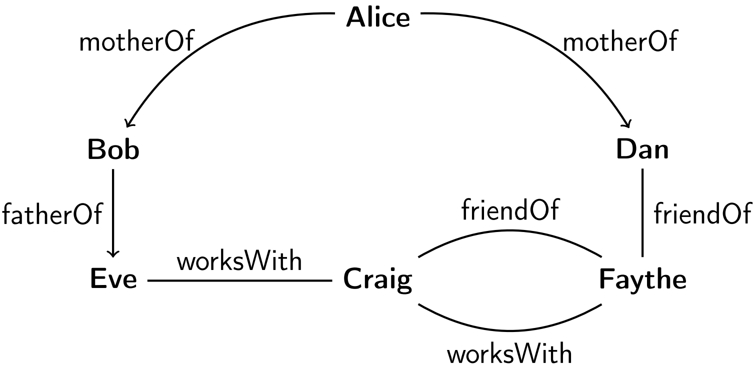
Querying graphs

$$\pi_1[\text{motherOf}] \circ [\text{motherOf} \cup \text{fatherOf}]^+$$



Querying graphs

$$\overline{\pi}_1[\mathsf{friendOf} \setminus \mathsf{worksWith}]$$



Navigational Expressions

$$e := \emptyset \mid \mathrm{id} \mid \ell \text{ (for } \ell \text{ an edge-label)} \mid e \circ e \mid e \cup e \mid$$
$$[e]^+ \mid \pi_1[e] \mid \pi_2[e] \mid \overline{\pi}_1[e] \mid \overline{\pi}_2[e] \mid e \cap e \mid e \setminus e$$

Question

Are all these operations necessary?
How does each operator influence expressive power?

Preliminary answer

We can use basic rewriting:

$$e_1 \cap e_2 = e_1 \setminus (e_1 \setminus e_2)$$
$$\pi_i[e] = \overline{\pi}_i[\overline{\pi}_i[e]]$$
$$\overline{\pi}_i[e] = \mathrm{id} \setminus \pi_i[e]$$

Problem statement

We start with $\{\emptyset, \mathrm{id}, \cup, \circ\}$ and edge-labels

- Add any subset $\mathfrak{F}$ of $\{\pi, \overline{\pi}, +, \cap, \setminus\}$,
  We denote the resulting query language by $\mathcal{N}(\mathfrak{F})$
- Compare the expressive power of resulting languages
- Graphs: already fully studied by Fletcher et al.
- Trees: a few results are known (XML)

Definition

Let $\mathfrak{F} \subseteq \{+, \pi, \overline{\pi}, \cap, \setminus\}$.
$\underline{\mathfrak{F}}$ is the superset of $\mathfrak{F}$ obtained by "basic rewriting".

Example

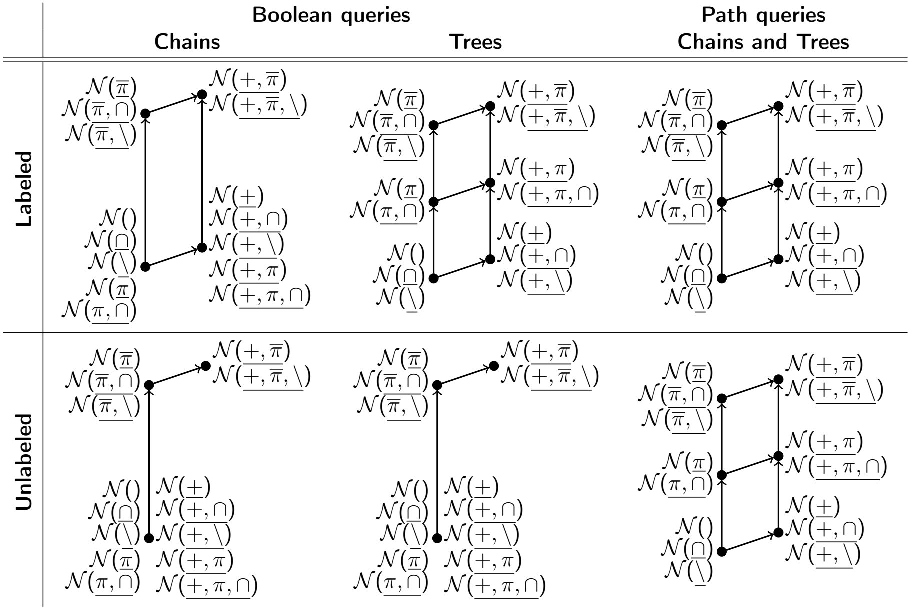
$\underline{\{\pi, \setminus\}} = \{\pi, \overline{\pi}, \cap, \setminus\}$

Overview

Results on trees and chains: the main results

- On labeled trees, we can do without $\cap$ and $\setminus$:
  we have $\mathcal{N}(\mathfrak{F}) \preceq_p \mathcal{N}(\underline{\mathfrak{F}} \setminus \{\cap, \setminus\})$
- For boolean queries:
  - On unlabeled trees, only $\overline{\pi}$ adds expressive power:
    we have $\mathcal{N}(+, \pi, \cap) \preceq_b \mathcal{N}()$
  - On labeled chains, we can do without π:
    we have $\mathcal{N}(+, \pi) \preceq_b \mathcal{N}(\mathfrak{F} \setminus \{\pi\})$

Results on trees and chains



Overview

The boolean collapse to $\mathcal{N}()$ on unlabeled trees

Theorem

Let $\mathfrak{F} \subseteq \{+, \pi, \cap\}$. On unlabeled trees we have $\mathcal{N}(\mathfrak{F}) \preceq_b \mathcal{N}()$.

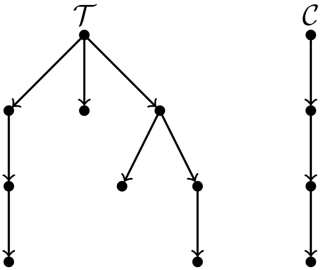
Definition (homomorphism)

A mapping $h : \mathbf{N}_1 \to \mathbf{N}_2$ is a *homomorphism* from $\mathcal{G}_1 = (\mathbf{N}_1, \mathcal{E}_1)$ to $\mathcal{G}_2 = (\mathbf{N}_2, \mathcal{E}_2)$ if $(m, n) \in \mathcal{E}_1$ implies $(h(m), h(n)) \in \mathcal{E}_2$.

Proposition

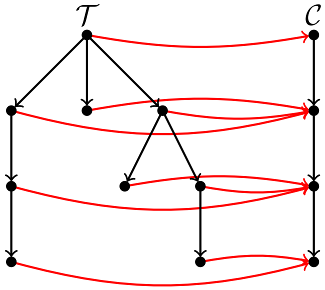
The language $\mathcal{N}(+, \pi, \cap)$ is closed under homomorphisms: if there is a homomorphism h from $\mathcal{G}_1$ to $\mathcal{G}_2$, then $h(e(\mathcal{G}_1)) \subseteq e(\mathcal{G}_2)$.

Proof: $\mathcal{N}(+, \pi, \cap)$ cannot distinguish trees from chains



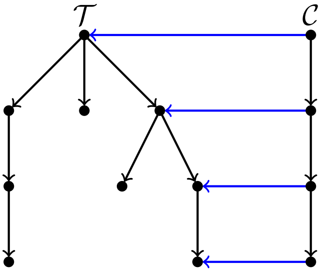
Proof: $\mathcal{N}(+, \pi, \cap)$ cannot distinguish trees from chains

- Provide homomorphism from tree $\mathcal{T}$ to chain $\mathcal{C}$



Proof: $\mathcal{N}(+, \pi, \cap)$ cannot distinguish trees from chains

- Provide homomorphism from tree $\mathcal{T}$ to chain $\mathcal{C}$
- Provide homomorphism from chain $\mathcal{C}$ to tree $\mathcal{T}$



Proof: $\mathcal{N}(+, \pi, \cap)$ can only query on depth

Conclusion

Even $\mathcal{N}(+, \pi, \cap)$ can only express queries of the form:

The height of the tree is at least k $\left(= \ell^k = \underbrace{\ell \circ \ldots \circ \ell}_{k \text{ terms}}\right)$

Theorem

Let $\mathfrak{F} \subseteq \{+, \pi, \cap\}$. On unlabeled trees we have $\mathcal{N}(\mathfrak{F}) \preceq_b \mathcal{N}()$.

Overview

Removing $\cap$ and $\setminus$ from simple queries

Example

$$\left[\ell^3\right]^+ \cap \left[\ell^7\right]^+ = \left[\ell^{21}\right]^+$$

$$\left[\ell^3\right]^+ \setminus \left[\ell^7\right]^+ = \left(\ell^3 \cup \ell^6 \cup \ell^9 \cup \ell^{12} \cup \ell^{15} \cup \ell^{18}\right) \circ \left(\left[\ell^{21}\right]^+ \cup \mathrm{id}\right)$$

Basic observations

- Expressions in $\mathcal{N}(+)$ are regular path queries
- Regular languages (expressions) are closed under $\cap$ and $\setminus$

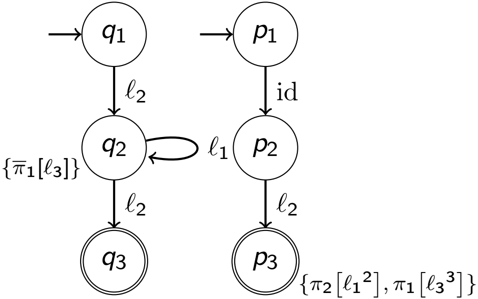
Question

How to generalize to π and $\overline{\pi}$?

Condition automata

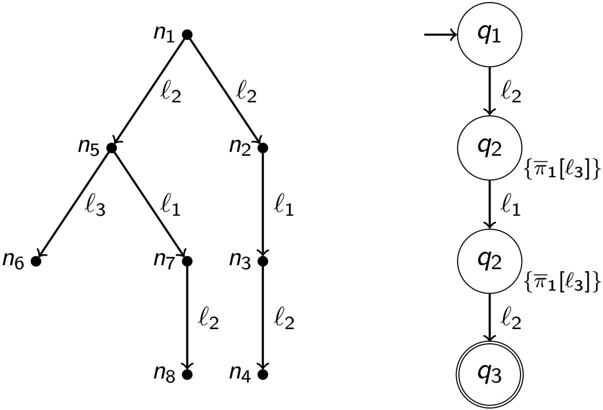
Definition (condition automaton)

A *condition automaton* is a 7-tuple $\mathcal{A} = (S, \Sigma, C, I, F, \delta, \gamma)$.



Semantics of condition automata

- Take a path in automaton from initial to final state
- Map to a path in a tree from m to n with equal labeling
- State s maps to node k: k satisfies the conditions of s



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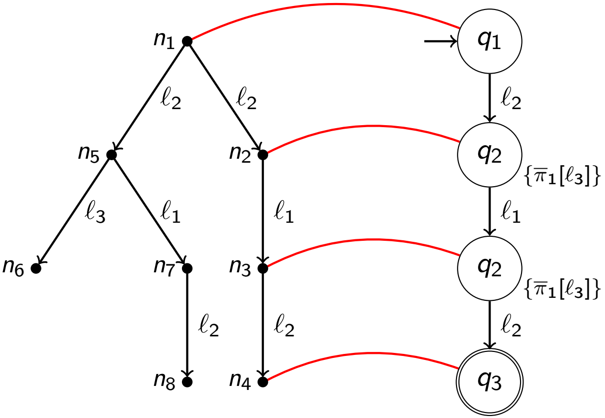

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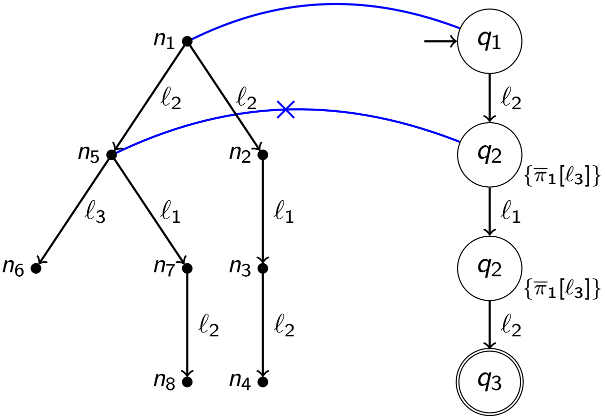


Condition automata and navigational expressions

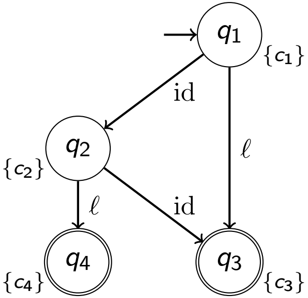
Proposition

Let $\mathfrak{F} \subseteq \{+, \pi, \overline{\pi}\}$. The class of condition automata specified for $\mathcal{N}(\mathfrak{F})$ in the following table is path-equivalent with $\mathcal{N}(\mathfrak{F})$.

Navigational language	Class of condition automata
$\mathcal{N}()$	$\{+, \pi, \overline{\pi}\}$-free and acyclic.
$\mathcal{N}(\pi)$	$\{+, \overline{\pi}\}$-free and acyclic.
$\mathcal{N}(\pi, \overline{\pi})$	$\{+\}$-free and acyclic.
$\mathcal{N}(+)$	$\{\pi, \overline{\pi}\}$-free.
$\mathcal{N}(+, \pi)$	$\{\overline{\pi}\}$-free.
$\mathcal{N}(+, \pi, \overline{\pi})$	no restrictions.

Condition automata and intersect

- Basically: use cross-product construction
- Take care of id-transitions by first removing them

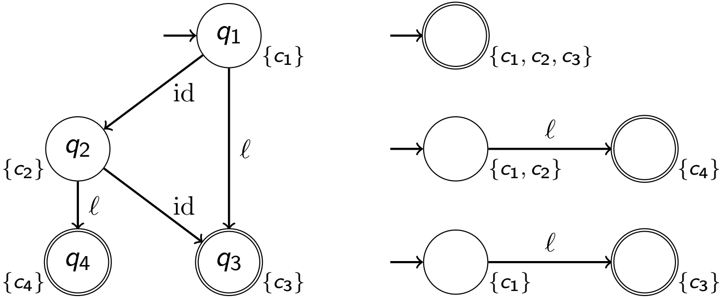


Proposition

Condition automata are closed under $\cap$.

Condition automata and intersect

- Basically: use cross-product construction
- Take care of id-transitions by first removing them



Proposition

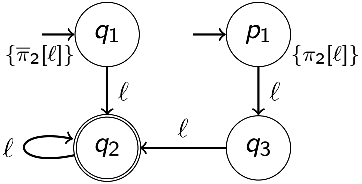
Condition automata are closed under $\cap$.

Condition automata and difference

- Difference: in terms of $\cap$ and complement: $S \setminus T = S \cap \overline{T}$
- In our setting: restrict $\overline{T}$ to the downward complement $\overline{T}_{\downarrow}$

Definition (deterministic condition automaton)

- For each node n: there exists exactly one initial state s such that n satisfies s.
- If node n satisifies state q, then, for each edge (n, ℓ, m) there exists exactly one transition (q, ℓ, p) such that p satisfies m.



Condition automata and downward complement

- Downward complement of deterministic condition automaton
  Swap the final states
- Conclusion: if we can construct a deterministic condition automaton, then condition automata are closed under $\setminus$

Proposition

For every condition automaton there is a path-equivalent deterministic condition automaton.

In this construction $\overline{\pi}$ is introduced if π was already used.

Theorem

On labeled trees we have $\mathcal{N}(\mathfrak{F}) \preceq_p \mathcal{N}(\underline{\mathfrak{F}} \setminus \{\cap, \setminus\})$.

Overview

Conclusions and Future Work

- Full characterization of the expressive power of downward navigational expressions
  - On trees and on chains
  - For boolean queries and path queries
- Typical non-downward operators are omitted
  - Next: we also include converse and diversity
  - We have some initial results

Results on graphs

Proposition (Fletcher et al. ICDT'11)

Let $\mathfrak{F}_1, \mathfrak{F}_2 \subseteq \{+, \pi, \overline{\pi}, \cap, \setminus\}$.

- *Labeled graphs:*
  - $\mathcal{N}(\mathfrak{F}_1) \preceq_b \mathcal{N}(\mathfrak{F}_2)$: *if* $\mathfrak{F}_1 \subseteq \underline{\mathfrak{F}_2}$.
- *Unlabeled Graphs:*
  - $\mathcal{N}(\mathfrak{F}_1) \preceq_p \mathcal{N}(\mathfrak{F}_2)$: *if* $\mathfrak{F}_1 \subseteq \underline{\mathfrak{F}_2}$.
  - $\mathcal{N}(\mathfrak{F}_1) \preceq_b \mathcal{N}(\mathfrak{F}_2)$: *if* $\mathfrak{F}_1 \subseteq \underline{\widetilde{\mathfrak{F}_2}}$ *or if*

$$\mathfrak{F}_1 \subseteq \{\pi\} \text{ and } \mathfrak{F}_2 = \mathfrak{F}_1 \cup \{+\}.$$